The Wilkie Way
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www.wilkieway.co.nz

Representing Equality

This months article draws on an article written by Charles Darr from NZCER, plus some of my own thoughts.
Firstly is it an equals sign or an equals symbol?
A sign - is something that can be read and interpreted - normally written in words
A symbol represents an idea, it's interpretation has been agreed upon by a social group. The same symbol can represent different ideas depending on the cultural context in which it is used. This can change over the course of history.

From a mathematical point of view, the equals symbol is not a command to do something. It is a signifier of a very important relationship – that of equality.
Equality is all about sameness. An equation is a special mathematical design that allows the user to describe the relationship that exists between expressions of equal value. In fact, the relationship of equality that the equals symbol defines is what allows equations to be manipulated and rearranged, so that new ways of expressing equality can be constructed and unknowns can be found. It makes equations the powerful tools that they are.
The "=" is a social convention that, over time, has been through a process of negotiation, modification and agreement. In fact, the pair of parallel lines we use today to signify that two expressions are equal has had a "bumpy" history. It is only in the last three hundred years or so that it has became generally recognised and used as the equals symbol.

So should we be concerned that so many students struggle with the meaning of the equals symbol?
The short answer is yes.
A weak sense of the equals symbol can and does affect learning in mathematics.
The research literature certainly emphasises this. Falkner (Falkner, Levi and Carpenter, 1999), for instance, comments that a non-mathematical sense of the equals symbol is "one of the major stumbling blocks for students when they move from arithmetic to algebra"

THE HISTORY OF THE EQUALS SYMBOL

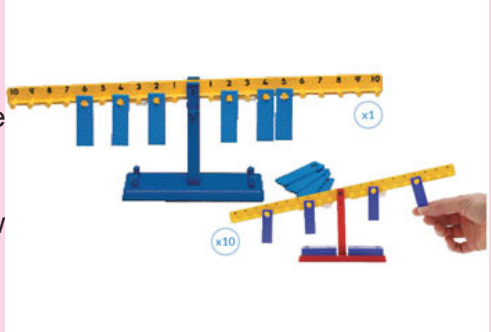
The equals symbol we employ today was first used to signify equality by an English mathematician, Robert Recorde, in a book he wrote about algebra called The Whetstone of Witte.
Recorde wanted to avoid having to "tediously repeat" the term "is equal to", so employed two parallel lines as a kind of shortcut His rationale for choosing to use two parallel lines was:
... bicause noe 2 thynges can be moare equalle.
At first the symbol was used inconsistently, sometimes even as a kind of decimal point. It was only from about Shakespeare's time, especially after it was employed in the work of such mathematical heavyweights as Descartes and Newton, that the equals symbol was adopted as the universal symbol to represent equality.

So how can we help our students to develop a richer mathematical sense of what "equals" means?
According to the literature, simply explaining what the symbol means is not enough. Falkner notes: "A concerted effort over an extended period of time is required to establish appropriate notions of equality" (Falkner, Levi and Carpenter, 1999, p.233). These writers also emphasise that it is important to start early: "Teachers should also be concerned about children's conceptions of equality as soon as symbols for representing number operations are introduced" (p.233). *Falkner, Levi & Karen, P. (1999). Children's understanding of equality foundation for algebra. Teaching Children Mathematics, 6 (4), 232-237.*

Some ideas to try in your classroom

Make the equals sign itself the subject of a conversation. There are various ways to do this. For instance, the symbol itself could be examined – perhaps even its history explored.

The equation as a balance Picturing equations as a kind of balance beam is another way to challenge students' ideas about the equals sign. Students can draw, imagine, or even use a pan balance to model how an equation works.



There is a piece of equipment called an equaliser - works like a balance beam, it has numbers 1 - 10 on either side of the balance and weight that can be hung to make the beam balance. For example if you put 4 and 5 on one side and 3 and 6 on the other side the beam will balance. The first school I taught in in NZ 20 years ago had 3 of these sitting on a shelf in a resource room because nobody knew what they were for. They very quickly made there way to my new entrant classroom.

Present students with equations such as n + 2 = 5 and ask them for any statements they can make about the value of the missing number, compared with 5. For instance, should it be bigger or smaller?

With older children, the numbers we use in the equations could be bigger, or decimals or fractions could be used. Older students could also be given equations such as x = y + 2 and asked to discuss what can be said about the number represented by the x, compared with the number represented by the y, if the equation is going to be true. *(You can use an empty square or square and circle instead of a letter to represent the missing number)*

Equations themselves can be used as discussion pieces.

When we present an equation such as 4 + 5 = n + 3 to a class or group, the different conceptions of the equals symbol that emerge can provide the basis for a productive session where children's ideas about equality can be expressed and considered.

In the year 7 PAT one of the questions is $4 + 5 = \square + 3$

The choices are: a. 6 b. 9 c. 12 d. 15

At an Intermediate School I was working in

10% of the students chose a. correct answer

65% of the students chose b. *4 + 5 = 9*

20% of the students chose c. *4 + 5 + 3 = 12*

remaining students guessed d.

One of the teachers conducted an small inquiry with her students writing equations on the board and asking for a show of hands if they agreed or disagreed they were valid equations.

6 + 3 = 9 100% 9 = 6 + 3 a few wavering in the belief you have to have the operation first

6 + 3 = 3 + 6 a few wavering but generally accepted because the numbers were just the opposite way round.

6 + 3 = 4 + 5 accepted by just a small handful of students.

When my first textbook was published my nephew (year 3) was very concerned that I had made a mistake. The equations he was concerned about were focused on the commutative property of addition

3 + 6 = 6 + ? and he knew 6 + 3 equalled 9.

I have often wondered how much students actually understood from our modelling of strategies like making double when the teacher happily records 8 + 6 = 7 + 7

What does the student actually see and understand from the recording?

There is definitely more to the equals sign than meets the eye. As we have seen, many students, even at Year 7 and 8, struggle to interpret its meaning.

Resources for Wilkie Way Members

Subscriptions purchased at the online store at www.wilkieway.co.nz

Individual $45 - paid via paypal

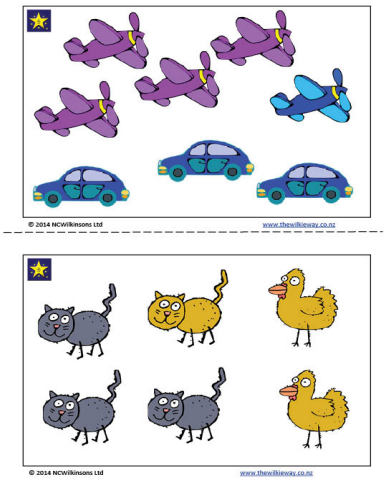
NZ School paid via invoice - complete form at online store

Up to 100 students $150+GST

101 - 300 students $250 + GST

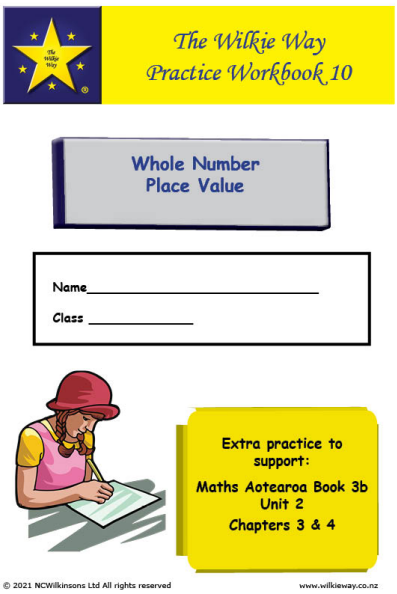
300+ students $350 +GST

Non NZ School $400 - paid via paypal



Foundation Resources

Equality pairs - eight pictures for discussion with number statement, two for each picture example: 5 and 3 for aeroplanes and cars and 4 and 4 for pink and blue.



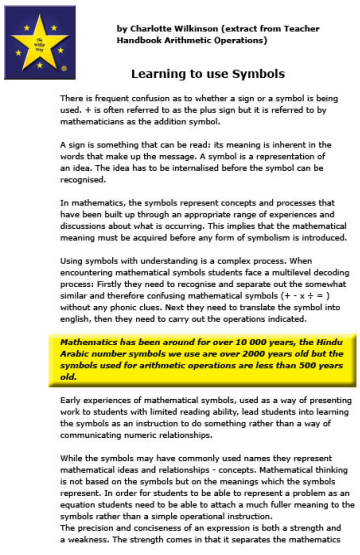
Maths Aotearoa Wookbooks Level 3b

Workbooks 9 - 16 for level 3

Teacher Professional Resources

Measurement and Geometric Progressions have been updated to include the links to the Maths Aotearoa resources

February Featured Resource



by Charlotte Wilkinson (extract from Teacher Handbook Arithmetic Operations)

Learning to use Symbols

There is frequent confusion as to whether a sign or a symbol is being used. + is often referred to as the plus sign but it is referred to by mathematicians as the addition symbol.

A sign is something that can be read: its meaning is inherent in the words that make up the message. A symbol is a representation of an idea. The idea has to be internalised before the symbol can be recognised.

In mathematics, the symbols represent concepts and processes that have been built up through an appropriate range of experiences and discussions about what is occurring. This implies that the mathematical meaning must be acquired before any form of symbolism is introduced.

Using symbols with understanding is a complex process. When encountering mathematical symbols students face a multilevel decoding process: Firstly they need to recognise and separate out the somewhat similar and therefore confusing mathematical symbols (+ - x ÷ =) without any phonic clues. Next they need to translate the symbol into english, then they need to carry out the operations indicated.

Mathematics has been around for over 10 000 years, the Hindu Arabic number symbols we use are over 2000 years old but the symbols used for arithmetic operations are less than 500 years old.

Early experiences of mathematical symbols, used as a way of presenting work to students with limited reading ability, lead students into learning the symbols as an instruction to do something rather than a way of communicating numeric relationships.

While the symbols may have commonly used names they represent mathematical ideas and relationships - concepts. Mathematical thinking is not based on the symbols but on the meanings which the symbols represent. In order for students to be able to represent a problem as an equation students need to be able to attach a much fuller meaning to the symbols rather than a simple operational instruction.
The precision and conciseness of an expression is both a strength and a weakness. The strength comes in that it separates the mathematics

Teacher Professional Resources
Professional Readings

Learning to Use Symbols - the background to the operation symbols and the equals symbols and learning to use them

The Wilkie Way

Problems with Equals

Mum gave Sam 6 lollies and Hira 10 lollies.
"That's not fair!" said Sam.
"Sort it out and make it fair." said Mum

How many lollies should Sam and Hira have each to be fair?

Auntie gave Aria $20 to spend on her birthday.
She bought a teddy, a bar of chocolate and a book. (all costing whole dollars) and spent all her money.
How much could the teddy the chocolate and the book have cost each?

How many different answers can you find?

Dad painted a 50m fence using 3 different colours. 24m red, 16m blue and the rest yellow.
The neighbours painted the other side of the same fence using 4 different colours.

How much yellow paint did dad use?
What lengths of each of four colours could the neighbour have used?
Find at least five different solutions. Show as equations.

Mandy's class were investigating how students travelled to school.
There are 256 students in the school.
18 students cycle. Twice as many come by car as walk. Four times as many come by bus as walk.

How many students travel by each of the ways of getting to school?