Teacher Professional Development

Mathematics Learning Progressions

(curriculumprogresstools.education.govt.nz)

for Teaching and Learning

Focus Area:

Learning Progression Framework (LPF) for Mathematics

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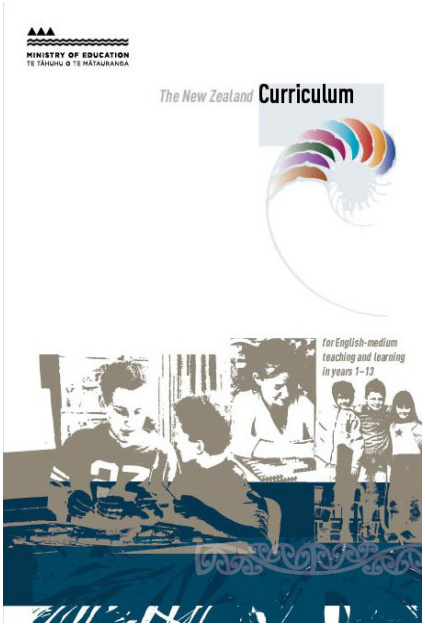
Teaching Mathematics In New Zealand Schools

There is only one mandated document which is the current curriculum document:



What is mathematics and statistics about?

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.

Mathematics is the exploration and use of patterns and relationships in quantities, space, and time. Statistics is the exploration and use of patterns and relationships in data. These two disciplines are related but different ways of thinking and of solving problems. Both equip students with effective means for investigating, interpreting, explaining, and making sense of the world in which they live.

Mathematicians and statisticians use symbols, graphs, and diagrams to help them find and communicate patterns and relationships, and they create models to represent both real-life and hypothetical situations. These situations are drawn from a wide range of social, cultural, scientific, technological, health, environmental, and economic contexts.

Why study mathematics and statistics?

By studying mathematics and statistics, students develop the ability to think creatively, critically, strategically, and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to enjoy intellectual challenge.

By learning mathematics and statistics, students develop other important thinking skills. They learn to create models and predict outcomes, to conjecture, to justify and verify, and to seek patterns and generalisations. They learn to estimate with reasonableness, calculate with precision, and understand when results are precise and when they must be interpreted with uncertainty. Mathematics and statistics have a broad range of practical applications in everyday life, in other learning areas, and in workplaces.

Learning area structure

The achievement objectives are presented in three strands. It is important that students can see and make sense of the many connections within and across these strands.

Number and algebra – Number involves calculating and estimating, using appropriate mental, written, or machine calculation methods in flexible ways. It also involves knowing when it is appropriate to use estimation and being able to discern whether results are reasonable. Algebra involves generalising and representing the patterns and relationships found in numbers, shapes, and measures.

Geometry and measurement – Geometry involves recognising and using the properties and symmetries of shapes and describing position and movement. Measurement involves quantifying the attributes of objects, using appropriate units and instruments. It also involves predicting and calculating rates of change.

Statistics involves identifying problems that can be explored by the use of appropriate data, designing investigations, collecting data, exploring and using patterns and relationships in data, solving problems, and communicating findings. Statistics also involves interpreting statistical information, evaluating data-based arguments, and dealing with uncertainty and variation.

Learning Progression Framework

What is it?

The mathematics framework provides a high-level map to describe what progress in mathematics and statistics looks like. It illustrates this progress by showing how students respond when problems become more complex and students need to apply their skills and knowledge in more complex ways.

This focus on solving problems does not negate the importance of students learning mathematical facts and procedures. It does, however, emphasise the pointlessness of knowledge without understanding and without the ability to apply that knowledge.

Where has it come from ?

The Ministry of Education convened workshops with mathematics and statistics experts (academics, researchers, and teachers) to discuss the ways in which mathematics should be organised for the Learning Progression Frameworks. The experts advised that the mathematics framework should be structured to further support the effective teaching of the mathematics and statistics learning area of the New Zealand Curriculum (NZC). They also advised that the framework should build on the research and experience of the Numeracy Development Projects. The proposed aspects for the mathematics framework were organised according to the strands of the mathematics and statistics learning area.

Following the initial workshops, the aspects were developed by mathematics educators according to their subject-matter expertise. As a result of developing the signposts for each of the aspects, the subject-matter experts proposed changes to four of the initially proposed aspects.

The two algebra-related aspects, proportional reasoning and algebraic thinking, were further considered by a researcher who had undertaken a significant piece of work developing an algebraic framework. This work pointed clearly to the fundamental importance of the patterns and relationships aspect, but also made evident the significance of using symbols and expressions to think mathematically across the whole learning area. This led to the two algebra-related aspects. It also resulted in proportional reasoning being omitted as a separate aspect in the framework. The rationale for this was that the Number Framework unnaturally separated proportional reasoning from the two pairs of reciprocal number operations, and in particular from multiplication and division. The decision was made to embed proportional reasoning within the multiplicative thinking aspect and the two algebra-related aspects.

The final eight aspects of the mathematics framework cover the breadth of the mathematics and statistics learning area and emphasise making sense of mathematical ideas and reasoning mathematically.
Four of the aspects address key ideas in the number and algebra strand, two address the geometry and measurement strand, and two address the statistics strand.

Organisation

Each progression is divided up into a series of signposts which reflect distinct changes in ways students are able to think using the knowledge they are building. These signposts do not directly match a curriculum level. Some curriculum levels have more than one signpost (especially at the younger end) and some signposts overlap two levels.
Neither progressions or levels have distinct edges. Student thinking does not progress at an even rate nor will students think the same every day or on every problem they meet. Progress is gradual (and messy) over time and with ongoing challenge and support from teachers to develop increasingly sophisticated thinking.

Supporting the Number & Algebra Curriculum Achievement Objectives

Additive thinking

This progression has eight signposts and is similar to the additive domain of the Number Framework in that it focuses on the increasingly sophisticated and flexible addition and subtraction strategies that students develop to solve increasingly complex problems. However, the sets of exemplars are not a direct match to the stages of the Number Framework. For example, imaging (stage 3 in the additive domain) is not identified by a discrete set of exemplars, and the higher stages of the domain are represented by more than one set of exemplars.

Level 1	Signposts 1 - 4	Level 2	Signposts 5 - 6	Level 3	Signposts 6 - 7
Level 4	Signpost 8	Level 5	No further signposts		

Multiplicative thinking

This progression has nine signposts and combines elements from both the multiplicative and proportional domains of the Number Framework.
However, as with additive thinking, the sets of exemplars are not a direct match to the stages of the Number Framework. For example skip counting (stage 4) and repeated addition (stage5) fit in the same signpost (Signpost 3) This progression focuses on students' ability to think multiplicatively as they solve multiplication, division, and proportional problems involving an extended range of whole numbers, decimals, fractions, ratios, and percentages, and in a range of contexts.

Level 1	Signposts 1 - 3	Level 2	Signposts 3 - 4	Level 3	Signposts 4 - 6
Level 4	Signpost 7 - 8	Level 5	Signposts 8 - 9		

Patterns and relationships

This algebraic thinking progression has seven signposts and develops students' understanding of the structure of and the relationships between numbers, shapes, and measures. In exploring patterns of increasing complexity, students develop the ability to recognise, explain, and generalise relationships between quantities and objects.

Level 1	Signposts 1 - 3	Level 2	Signposts 3 - 4	Level 3	Signposts 4 - 5
Level 4	Signpost 5 - 6	Level 5	Signposts 6 - 7		

Using symbols and expressions to think mathematically

This algebraic thinking progression has seven signposts and is fundamental to all other aspects of mathematics. It focuses on the ways in which symbols, expressions, and equations are used to communicate mathematical ideas.
In solving problems in a range of contexts, students must make sense of the symbols they read and must be able to express their understanding of a problem using the symbolic language of mathematics.

Level 1	Signposts 1 - 2	Level 2	Signposts 3 - 4	Level 3	Signposts 4 - 5
Level 4	Signpost 5 - 6	Level 5	Signposts 6 - 7		

This aspect of mathematics while fundamental to all other progressions is also likely to be the most neglected in a teaching programme.

Symbolic language: A symbol is a representation of an idea. The idea has to be internalised before the symbol has meaning. In mathematics, the symbols represent concepts and processes that have built up through an approportiate range of experiences and dicussions about what is occuring. This implies that mathematical meaning must be acquired before any form of symbolism is introduced.

Using symbols as a way of presenting work to students with limited reading ability leads students into learning the symbols as an instruction to do something rather than as a way of communicating numeric relationships. This will then have an ongoing impact on their future mathematical learning.

The need to develop the language of mathematics is clearly highlighted in the curriculum document (page 16)

Learning areas and language
Each learning area has its own language or languages.
As students discover how to use them, they find they are able to think in different ways, access new areas of knowledge, and see their world from new perspectives.

For each area, students need specific help from their teachers as they learn:
• the specialist vocabulary associated with that area;
• how to read and understand its texts;
• how to communicate knowledge and ideas in appropriate ways;
• how to listen and read critically, assessing the value of what they hear and read.

In addition to such help, students who are new learners of English or coming into an English-medium environment for the first time need explicit and extensive teaching of English vocabulary, word forms, sentence and text structures, and language uses.
As language is central to learning and English is the medium for most learning in the New Zealand Curriculum, the importance of literacy in English cannot be overstated.

Supporting the Geometry and Measurement Curriculum Achievement Objectives

Geometric thinking

The geometric thinking aspect was challenging because the progression needed to describe increasing sophistication in three related but different elements of geometry: shape, transformation, and position and orientation. This meant that the aspect, through the sets of illustrations, needed to describe developing expertise in each of the three elements. In addition, less is known about the development of geometric expertise in primary education. For example, the most cited framework of geometric thinking (van Hiele) describes five levels of thinking, but only two of these are applicable to levels one to four of the NZC. These two factors resulted in the identification of fewer signposts than the other aspects.

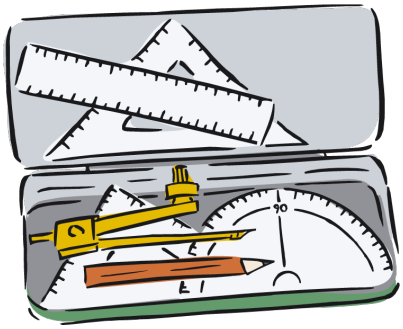
This progression has six signposts. As students make sense of and navigate their spatial world, they come to recognise, describe, and use the properties and symmetries of shapes, and they describe movement and position with increasing accuracy.

Level 1	Signposts 1 - 2	Level 2	Signpost 3	Level 3	Signpost 4
Level 4	Signpost 5	Level 5	Signpost 6		

Measurement sense

This progression has eight signposts and is built on established learning progressions that start from identifying the attribute (for example, length or weight) and progress through the use of non-standard measurement units to the use of standard measurement units and formulae.
Understanding what a measurable attribute is and becoming familiar with the units, systems, and processes that are used in measuring attributes is the focus of this progression. Progression in the understanding of measurement is determined by increased sophistication in the measurable attributes of objects and the complexity of the attribute being measured.

Level 1	Signposts 1 - 3	Level 2	Signposts 3 - 4	Level 3	Signposts 5 - 6
Level 4	Signpost 6 - 7	Level 5	Signposts 7 - 8		



Supporting the Statistics Curriculum Achievement Objectives

The two statistics aspects in the curriculum document; statistical thinking and statistical literacy, were reviewed by a small team with expertise in this area. Their work made clear the distinction between students' enactment of the statistical investigations cycle, and students' ability to interpret their own and others' data as well as chance situations. This led to the two final progressions.

Statistical investigations

The statistical investigations progression has six signposts and is based on the students' development of an increasingly sophisticated implementation of the statistical inquiry cycle that includes posing investigative questions, collecting data, displaying data, and discussing results.

Level 1	Signposts 1 - 2	Level 2	Signpost 3	Level 3	Signposts 4 -5
Level 4	Signpost 5 - 6	Level 5	Signpost 6		

Interpreting statistical and chance situations

This progression has five signposts. As students are exposed to the statistical evidence presented by others, they need to be able to interpret and gain information from what they see, critically evaluating both the quality of the evidence and the arguments being presented on the basis of that evidence.

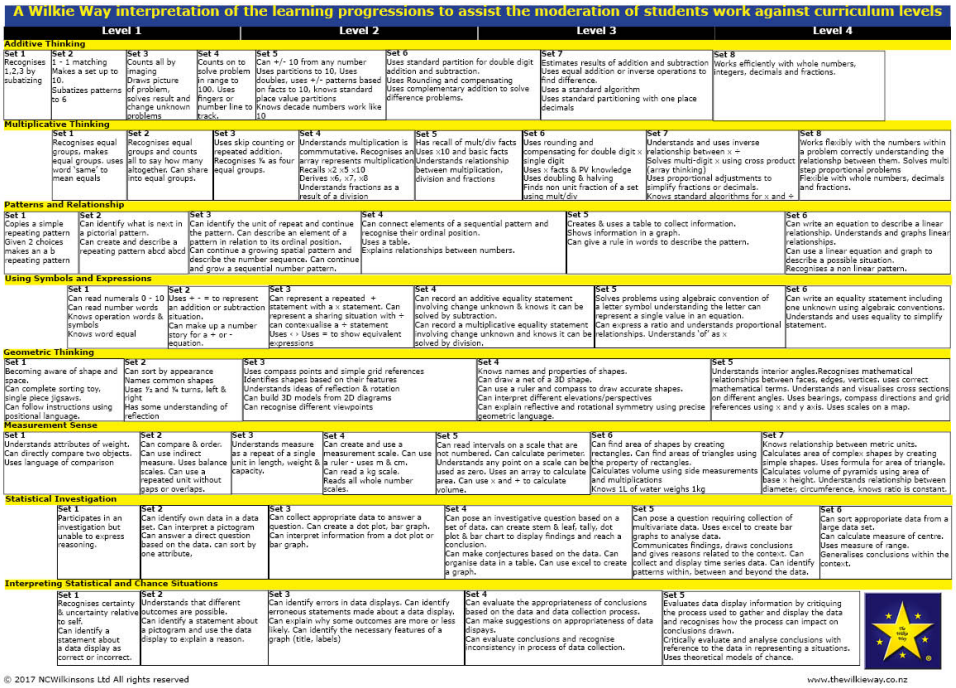
Level 1	Signposts 1 - 2	Level 2	Signpost 3	Level 3	Signposts 4
Level 4	Signpost 4 - 5	Level 5	Signpost 5		

Using the exemplars

The exemplars for the learning progressions which can be found on the curriculum progress tools website https://curriculumprogresstools.education.govt.nz/lpf-tool/ cannot give a full picture of everything that should be in a comprehensive mathematics programme but assist in making summative assessment decisions about a student.

They are exemplars to demonstrate what type of activities you may use and what a students thinking looks like at a particular signpost.

A chart showing what each exemplar represents can be found in the members area of wilkieway.co.nz

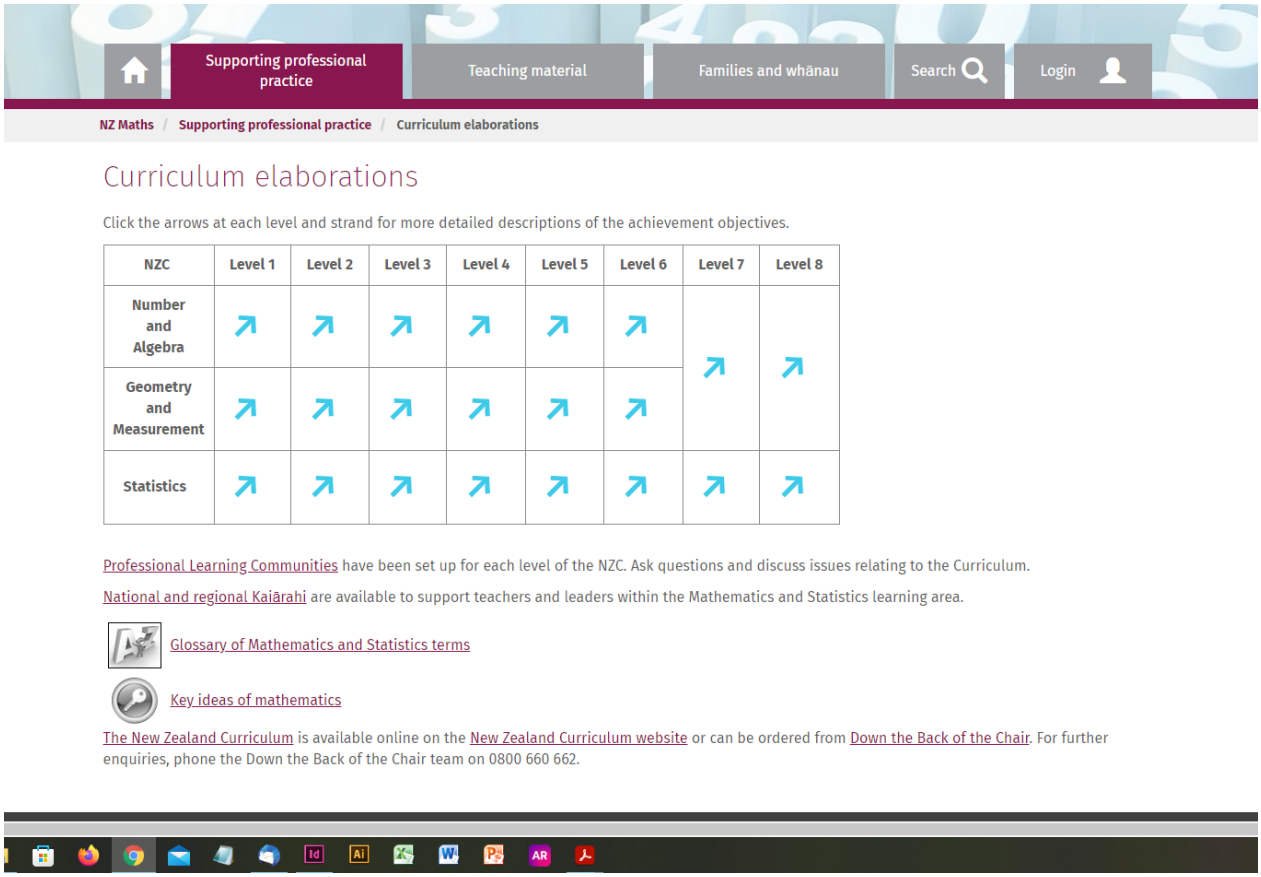


A Wilkie Way interpretation of the learning progressions to assist the moderation of students work against curriculum levels

	Level 1	Level 2	Level 3	Level 4
Additive Thinking	Set 1 Recognises 1,2,3 by subitizing Set 2 1 - 1 matching Makes a set up to 10. Subatizes patterns to 6 Set 3 Counts all by imaging Draws picture of problem, solves result and change unknown problems Set 4 Counts on to solve problem in range to 100. Uses Fingers or number line to track.	Set 5 Can +/- 10 from any number Uses partitions to 10. Uses doubles, uses +/- patterns based on facts to 10, knows standard place value partitions Knows decade numbers work like 10 Set 6 Uses standard partition for double digit addition and subtraction. Uses Rounding and compensating Uses complementary addition to solve difference problems.	Set 7 Estimates results of addition and subtraction Uses equal addition or inverse operations to find difference. Uses a standard algorithm Uses standard partitioning with one place decimals	Set 8 Works efficiently with whole numbers, integers, decimals and fractions.
Multiplicative Thinking	Set 1 Recognises equal groups, makes equal groups. uses word 'same' to mean equals Set 2 Recognises equal groups and counts all to say how many altogether. Can share into equal groups. Set 3 Uses skip counting or repeated addition. Recognises ¼ as four equal groups.	Set 4 Understands multiplication is commmutative. Recognises an array represents multiplication Recalls x2 ,5 ,x10 Derives x6, x7, x8 Understands fractions as a result of a division Set 5 Has recall of mult/div facts Uses x10 and basic facts Understands relationship between multiplication, division and fractions	Set 6 Uses rounding and compensating for double digit x single digit Uses ½ facts & PV knowledge Uses doubling & halving Finds non unit fraction of a set using mult/div Set 7 Understands and uses inverse relationship between x ÷ Solves multi-digit x using cross product (array thinking) Uses proportional adjustments to simplify fractions or decimals. Knows standard algorithms for x and ÷	Set 8 Works flexibly with the numbers within a problem correctly understanding the relationshp between them. Solves multi step proportional problems Flexible with whole numbers, decimals and fractions.
Patterns and Relationship	Set 1 Copies a simple repeating pattern Gives 2 choices makes an a b repeating pattern Set 2 Can identify what is next in a pictorial pattern. Can create and describe a repeating pattern abcd abcd	Set 3 Can identify the unit of repeat and continue the pattern. Can describe an element of a pattern in relation to its ordinal position. Can continue a growing spatial pattern and describe the number sequence. Can continue and grow a sequential number pattern. Set 4 Can connect elements of a sequential pattern and recognise their ordinal position. Uses a table. Explains relationships between numbers.	Set 5 Creates & uses a table to collect information. Shows information in a graph. Can give a rule in words to describe the pattern.	Set 6 Can write an equation to describe a linear relationship. Understands and graphs linear relationships. Can use a linear equation and graph to describe a possible situation. Recognises a non linear pattern.
Using Symbols and Expressions	Set 1 Can read numerals 0 - 10 Can read number words Knows operation words & symbols Knows word equal Set 2 Uses + - = to represent an addition or subtraction situation. Can make up a number story for a + or - equation.	Set 3 Can represent a repeated + statement with a x statement. Can represent a sharing situation with ÷ can conceualise a ÷ statement Uses ‹ › Uses = to show equivalent expressions Set 4 Can record an additive equality statement involving change unknown & knows it can be solved by subtraction. Can record a multiplicative equality statement involving change unknown and knows it can be solved by division.	Set 5 Solves problems using algebraic convention of a letter symbol understanding the letter can represent a single value in an equation. Can express a ratio and understands proportional relationships. Understands 'of' as x	Set 6 Can write an equality statement including one unknown using algebraic conventions. Understands and uses equality to simplify statement.
Geometric Thinking	Set 1 Becoming aware of shape and space. Can complete sorting toy, single piece jigsaws. Can follow instructions using positional language. Set 2 Can sort by appearance Names common shapes Uses ¼ and ½ turns, left & right Has some understanding of reflection	Set 3 Uses compass points and simple grid references Identifies shapes based on their features Understands ideas of reflection & rotation Can build 3D models from 2D diagrams Can recognise different viewpoints Set 4 Knows names and properties of shapes. Can draw a net of a 3D shape. Can use a ruler and compass to draw accurate shapes. Can interpret different elevations/perspectives Can explain reflective and rotational symmetry using precise geometric language.	Set 5 Understands interior angles.Recognises mathematical relationships between faces, edges, vertices, uses correct mathematical terms. Understands and visualises cross sections on different angles. Uses bearings, compass directions and grid references using x and y axis. Uses scales on a map.	
Measurement Sense	Set 1 Understands attributes of weight. Can directly compare two objects. Uses language of comparison Set 2 Can compare & order. Can use indirect measure. Uses balance scales. Can use a repeated unit without gaps or overlaps. Set 3 Understands measure as a repeat of a single unit in length, weight & capacity.	Set 4 Can create and use a measurement scale. Can use a ruler - uses m & cm. Can read a kg scale. Reads all whole number scales. Set 5 Can read intervals on a scale that are not numbered. Can calculate perimeter. Understands any point on a scale can be used as zero. Uses an array to calculate area. Can use x and + to calculate volume.	Set 6 Can find area of shapes by creating rectangles. Can find areas of triangles using the property of rectangles. Calculates volume using side measurements and multiplications Knows 1L of water weighs 1kg Set 7 Knows relationship between metric units. Calculates area of complex shapes by creating simple shapes. Uses formula for area of triangle. Calculates volume of pyramids using area of base x height. Understands relationship between diameter, circumference, knows ratio is constant.	
Statistical Investigation	Set 1 Participates in an investigation but unable to express reasoning.	Set 2 Can identify own data in a data set. Can interpret a pictogram Can answer a direct question based on the data, can sort by one attribute. Set 3 Can collect appropriate data to answer a question. Can create a dot plot, bar graph. Can interpret information from a dot plot or bar graph. Set 4 Can pose an investigative question based on a set of data, can create stem & leaf, tally, dot plot & bar chart to display findings and reach a conclusion. Can make conjectures based on the data. Can organise data in a table. Can use excel to create a graph.	Set 5 Can pose a question requiring collection of multivariate data. Uses excel to create bar graphs to analyse data. Communicates findings, draws conclusions and gives reasons related to the context. Can collect and display time series data. Can identify patterns within, between and beyond the data. Set 6 Can sort appropriate data from a large data set. Can calculate measure of centre. Uses measure of range. Generalises conclusions within the context.	
Interpreting Statistical and Chance Situations	Set 1 Recognises certainty & uncertainty relative to self. Can identify a statement about a data display as correct or incorrect.	Set 2 Understands that different outcomes are possible. Can identify a statement about a pictogram and use the data display to explain a reason. Set 3 Can identify errors in data displays. Can identify erroneous statements made about a data display. Can explain why some outcomes are more or less likely. Can identify the necessary features of a graph (title, labels)	Set 4 Can evaluate the appropriateness of conclusions based on the data and data collection process. Can make suggestions on appropriateness of data dispays. Can evaluate conclusions and recognise inconsistency in process of data collection. Set 5 Evaluates data display information by critiquing the process used to gather and display the data and recognises how the process can impact on conclusions drawn. Critically evaluate and analyse conclusions with reference to the data in representing a situations. Uses theoretical models of chance.	

www.thewilkieway.co.nz

To develop a comprehensive mathematics programme for your students you will need to have an indepth understanding of the extent of the achievement objectives as set out in the curriculum document. Go to the curriculum elaborations on NZ Maths, https://nzmaths.co.nz/curriculum-elaborations



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Curriculum elaborations

Click the arrows at each level and strand for more detailed descriptions of the achievement objectives.

NZC	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6	Level 7	Level 8
Number and Algebra	↗	↗	↗	↗	↗	↗	↗	↗
Geometry and Measurement	↗	↗	↗	↗	↗	↗		
Statistics	↗	↗	↗	↗	↗	↗	↗	↗

Professional Learning Communities have been set up for each level of the NZC. Ask questions and discuss issues relating to the Curriculum.

National and regional Kaiārahi are available to support teachers and leaders within the Mathematics and Statistics learning area.

Glossary of Mathematics and Statistics terms

Key ideas of mathematics

The New Zealand Curriculum is available online on the New Zealand Curriculum website or can be ordered from Down the Back of the Chair. For further enquiries, phone the Down the Back of the Chair team on 0800 660 662.

Mathematics Learning Progression Signpost Alignments