Teaching and Learning Level 1 Mathematics

Level 1 Maths has been broken into 8 large blocks of work (units) to create the necessary building up of knowledge and skills. Each block being reliant on the previous block of learning.

Each unit is made up of 6 elements ensuring content across all strands is covered. The best practice for delivery of content should be through a mixture of explicit teaching, guided practice, flexible grouping and independent activity, including play based activities with interactive dialogue to develop deeper thinking and language development in meaningful contexts. The elements within a unit are interconnected so it is important that connections are made to develop a robust foundational understanding of mathematics not just number, or space or measurement or statistics.

The first two units are the essential foundation blocks for mathematical success. Some students on entry to school will need to start right at the beginning, some will start at the beginning and make very fast progress, some will already have significant knowledge and understanding. Undertaking a baseline assessment is useful and especially so to identify those students with a lot of knowledge and those whose progress requires more support.

A baseline assessment is available to Wilkie Way members under the heading Assessment in the resource directory.

Maths Aotearoa Book 1A and Book 1B has suggested classroom activities, many using equipment readily available in a junior classroom. There are also a total of 200 activity cards supporting the units. Many of the activities are practical and can be used multiple times. Some have activities on both sides of the cards and some have teacher information for further teacher guidance in developing the mathematics from the activity.

Further practice for each unit can be found in 25 printable number and algebra workbooks and 4 measurement workbooks and teacher notes available to Wilkie Way members.

Maths Aotearoa Book 1A

Unit 1 - Making Sense of Small numbers:

Counting, Sequencing & Patterning	Reading Numerals & Making groups up to 6	Comparing & Ordering numbers to 6
Statistics Sorting & Matching	Working with Shapes Continued through Unit 2	Describing Position Continued through Unit 2

From the outset of a student's journey in learning mathematics teachers must keep in mind the essence of mathematics is in the patterns and relationships.

"What is mathematics and statistics about?

Mathematics is the exploration and use of patterns and relationships in quantities, space and time. Statistics is the exploration and use of patterns and relationships in data."

Seeking patterns and relationships requires students to identify and notice attributes and whether they are the same or different. They will also need to consider if the attributes noticed relate to each other and how they relate.
This noticing occurs pre-number (starting from the moment a child is born) as they consider visual attributes. With increasing knowledge and language they are able to verbalise for example colour and shape. The attributes may have to do with measure – size, mass etc. and a substantial language base is

required to support the developing concepts and knowledge.

Many of a student's early experiences are in the geometric strand of the mathematics curriculum through their interactions with the world around them. Making sense of the geometric elements requires development of language to describe, explain and think.

Students develop an understanding of numbers through real experiences.
Humans have counted and compared quantities for a very long time (since the beginnings of civilisations). The ability to recognise the difference in quantity between small numbers (one, two and three) without counting is called subitizing. Much research suggest this ability can be found in very young babies but further research also suggests there is a developmental brain maturation element to this as well. The picture remains complicated and the complexity in developing number understanding requires areas of the brain to work together. Visualisation and symbolic representations of numbers develop interdependently. Variations in experiences, including language, counting practice, and representations seem likely to affect the development.

We have a distinct set of number words in a particular order that need to be mastered. There is a big difference between being able to say the number words in sequence and understanding numbers.
A number is an abstract idea and to be a number it must be a quantity of something. Students need to come to the understanding that the counting words are assigned to an individual item and the last number in the counting set tells you how many there are. To achieve this, a student must learn the names for the numbers and the order in which the words come. They must use a one to one relationship (one word to one item) and learn to recognise the symbol that represents the counting word and the quantity in the group of objects. This is the cardinal aspect of number.

They must also begin to consider the position of individual numbers in a sequence of numbers using language of before, after, between, first, second, third. This is a more abstract idea and is the ordinal aspect of number. It is reliant on an understanding of sequencing which begins pre-number and is essential to reading and writing as well as mathematics. Much of what we do occurs in a sequence of events – for example getting dressed, daily routines.
To assist the development of ordinal number, to focus students on the position of the number word in the sequence, items to be counted should be arranged in a row (or column). When a student counts along a row he/she is not only setting up a one – one correspondence with the set of number names, he/she is also naming each item by its position in the row. Number two is the second in the row, number three is the third. When the student is counting he/she is not necessarily thinking of the set of three items he/she has counted but that the item is the third in the row. For the ordinal aspect of number to be present in the activity, the things counted must be seen in order.

Many young children struggle with sequencing and it certainly has a developmental aspect. Visual sequencing can be slow to develop but for most students just requires lots of opportunities to practice.

Unit 2 Exploring Numbers to 10

Numerals & Making Groups to 10	Compare and Order Numbers to 10	Joining & Partitioning Sets of Objects
Exploring Groupings within 5	Working with Shapes Continued from Unit 1	Describing Position Continued from Unit 1

Students have an idea that numbers are used for counting sets of objects. They extend their numeral identification to all digits 0 – 9 including the first combination of digits, number 10.

Their understanding of numbers expands from the natural numbers (counting numbers) to the whole numbers, which now includes zero. For zero to be a number it must represent nothing of something. If students see zero as just nothing then they will often disregard it as not important.

Number 10 should be identified as a special number from the outset even though students will not yet be able to use a set of 10 as a counting unit with understanding. The decade names should be modelled and used in the classroom,

- Keeping a record of the number of days in school by adding a lollipop stick to a pot and when a group of ten is made move it to the next pot. Keep a number chart to show the number of groups of ones and the number of groups of ten.
- Making reward sticker charts in groups of 10.
- Create opportunities to count in groups of ten where the ten ones are evident.

This work is helping to build vocabulary. There are 29 number names to master in order to count to 100.

Along with language building they are developing their understanding of the cardinal and ordinal aspects of number. Work on positional language continues as in Unit one and students should begin connecting the idea of one more with the number after and one less or fewer with the number before in the counting sequence in ones.
As they learn to quantify, noticing “how many” becomes another attribute for sorting: more, less, equal group.
Grammatically “fewer” can only be used when referring to countable objects – “less” is often used for both countable objects and singular mass nouns (money, love etc.). Students need to have the words fewer and fewest in their vocabulary as they will meet the words during their mathematics learning journey.
Likewise “How much?” is only used for singular mass nouns while “how many?” is only used for countable nouns – a very common grammatical error which should not go uncorrected.

Sorting shapes should now also include the quantifiable aspects of the shapes. For example

- The number of straight sides
- The number of corners
- The number of squares that can be counted on a cube

Partitioning sets begins the whole/part/part structure which underpins the inverse relationship between addition and subtraction. Students need to see that a whole number is made up of smaller whole numbers before they can combine whole numbers to make a bigger whole number. From these experiences they are being exposed to the commutative property of addition.

Special attention is given to groupings with five for students to focus on beginning to combine small numbers to make a bigger number and recognise the patterns within pairs to make the same number. The idea that a number can be made of different combinations helps build conservation of number. The understanding that the count of all objects in a set, in any order will always give the same number and the last number spoken in the sequence tells you how many. These patterns observed in these activities are the beginnings of commutative addition and understanding the next counting number is the number after and is the result of adding one more to the set. The number before is the result of taking one away from the set. These activities also begin the understanding of the inverse relationship between addition and subtraction, if I take one (subtract) from one part and give it (add) it to the other part the total amount does not change.

Units 3 and 4 begin the journey of operations with numbers - addition, subtraction, multiplication and division. Linear measure activities connect to developing concepts of addition and subtraction and the statisitics and probability elements which continue through both units 3 and 4 extend student number knowledge by developing early multiplicative ideas about numbers.

Unit 3: Combining, Comparing and Ordering

Addition & Subtraction	Attribute of Measure	Comparing Measure
Shape & Pattern	Statistics Continued through Unit 4	Probability Continued through Unit 4

With increasing number knowledge, counting can be used to describe shapes more accurately by using number of sides, number of corners and defining features using increasing geometric language. Consideration should be given to the purpose of particular shapes in our everyday lives. This is important to design and technology. Does the shape roll, stack or slide? When describing shapes consider the attributes a shape does not have. Learning to consider the negative attributes can be quite difficult for young students who are reliant on what they can see. However much of mathematics is abstract and students need to learn to image what is and what isn't to develop spatial and numeric reasoning. Student/teacher dialogue should expand to encourage spatial thinking.

Spatial thinking is essential to problem solving. A visual pictorial mode of thought enables a student to visualise a problem geometrically – hence the problem solving strategy of draw a picture.

Tessellating patterns introduces the idea of covering a surface with no gaps or overlaps which will eventually lead to measurement of area.

Linear measure underpins ideas about addition and subtraction in a very meaningful context. A bar model, using materials such as Cuisenaire or number strips representing addition and subtraction is joining, separating, comparing and ordering units of length.

The concepts of measure are the same for whatever is being measured. Students need to consider what can be measured: Length, mass, capacity, volume, area, temperature, angle, time. There is an enormous amount of vocabulary associated with concepts of measure that students need to develop as their ideas about measurement evolve. At this level students are identifying the attributes that can be measured and then making comparisons in practical situations with appropriate dialogue focusing on developing the associated vocabulary. Direct comparison as used in measurement is also the basis for additive comparison or finding the difference in quantity.

Time and in particular the passage of time is a difficult concept for children to judge. It should be taught in association with the specific times relevant to the students' lives. Again there is an extensive amount of vocabulary associated with time concepts. Learning in this area is very much based on experience and use must be made of every learning opportunity that arises during the school day to support understanding of time.

The statistical display of a uniform pictograph with objects allows for the direct comparison of quantities, including a set of zero in a meaningful situation. Direct comparison statements need to be asked of the data like:

- How many?
- Which set has more than another set?
- Which set has fewer/less than another set?

- Which set has the most?
- Which set has the least/fewest?

From a uniform pictograph students should be able to answer the questions without counting the objects but by making a measurement comparison. Focusing on the comparison focuses the thinking on the difference between quantities and introduces questions like how many more and how many less.

Introduce vocabulary of chance. Students need to realise that while there are many ways of partitioning a number, there is a finite number of pairs to make each number. These are known as the basic facts. While learning basic facts is very important it should not become the only focus of primary mathematics and used as a measure for mathematical success.

Addition and subtraction concepts continue to develop as number knowledge increases and students should be beginning to develop a solid understanding of the cardinality of numbers (number conservation). Students who have not attained the cardinal principal will be delayed in their ability to add and subtract with meaning. It would be hard to move them from counting all the objects when left to their own devises although they can learn the “trick” of counting on as a procedure under teacher guidance and if they know the number after the given number.

With some conceptual understanding of addition and subtraction it is appropriate to introduce the symbols representing the concepts by modelling their use in meaningful contexts. A meaningful concept requires the use of a story problem. Initially these problems will be verbal but using simple words and picture clues, students need to be taught how to approach the reading of mathematics.
Reading mathematics from the outset requires comprehension of the problem. Students should be encouraged to draw a simple picture of the problem before focusing on the symbolic equation for the situation. If students are only encouraged to write the equation for the problem they will only focus on the numbers and key words to give a clue to whether the problem is an addition or a subtraction. While at this stage the problems are straight forward, if students have not learnt how to read mathematics their problem solving later in their learning journey will be severely compromised.

Unit 4: Combining, Grouping and Sharing

Numbers to 20	Equal Grouping	Equal Sharing
Patterns & Symmetry	Statistics Continued from Unit 3	Probability Continued from Unit 3

Some students will already know the counting sequence to 20 and beyond but this unit is looking more closely at the teen words and the ty words. For many students the similarity in sound of these words leads to a teen/ty confusion. Connecting the word, symbol and representation of the quantity in a student’s mind is essential. This will only occur through multiple experiences of seeing the difference in material representation of teen numbers and ty numbers. Using Te Reo Maori emphasises the tens and ones structure of the teens numbers and the number of groups of ten for ty number.

A series of lessons exploring teens and tys is available under the Place Value heading in the Wilkie Way membership directory. Also resources for activities for understanding the teen and ty number symbols. The beginnings of place value are understanding the value of the digit in its position (place) in a number.

An understanding of quantity provides the foundation for understanding multiplication. A quantity is a characteristic of objects that can be counted or measured and consists of a number and a unit. The number words describe the number portion of a quantity and can be shown with representations, for example five counters. The number on its own does not exist. It can only be described in relation to something, e.g the counters. A count is a discrete quantity that answers the question “How many?”

Counting has begun with counting in ones and understanding the count is the cardinal principle of number. Students who have achieved or are close to achieving cardinality of number and have gained the knowledge of the number after in a sequence will be beginning to count on to solve addition type problems or use recall of known addition facts. Counting backwards to solve subtraction problems requires not only the knowledge of the number before but also a firm knowledge of the backward counting sequence. To use recall of subtraction facts students need to have knowledge of and apply the inverse relationship between addition and subtraction.

Understanding counting now progresses to counting in equal size groups larger than one.
Learning to rote count in twos, fives and tens does not mean students have an understanding of equal grouping. Experiences of counting objects in groups other than one is required. As students use subitizing small groups to make a count of items around them, they acquire the skill of counting in twos with an understanding that items number one, three, five etc are still present in the count.

Developing the counting sequences in twos, threes, fives and tens is necessary before students can use these sequences to solve equal grouping type problems.
From the experiences of counting items in equal groups, of adding equal groups the ideas of multiplication begin to develop. The first multiplication facts a student recalls will be the doubles but these are recorded at this stage as addition facts.

A very clear understanding of equal grouping and learning to use an equal group as a counting unit is essential to working with the equal group of ten. Work on place value, considering the value of the digit in the place it occupies in a number requires the use of a group of ten as a counting unit that behaves in the same way as groups of one but in a different place (column) in any number.

Division is the inverse operation of multiplication. Young students first use the idea of division at a very early stage, well before the concept of multiplication has been developed to any degree. A student uses two fundamentally different types of division. One is the act of equal sharing. While this sharing or partitioning type of division is mathematically more complex, students often use it earlier as no counting is needed to share out a non-counted set of items (bag of lollies) between an uncounted group of children. The sharer just has to keep going around the group giving one at a time until no lollies are left. It does not become numerical until specific numbers are given For example: Share out a packet of ten cookies between a group of five students.

The second type is when he/he works out how many students could have two cookies each from a packet of ten cookies. This aspect of division, grouping or quotition is mathematically simpler, but seems at first to young students to be completely unrelated to the equal sharing.

Early experiences of division, sharing and grouping must be set in practical situations so that an appropriate method for solving the problems can be used. Use equal sharing for sharing problems, and equal grouping for grouping problems.

While students have experienced the idea of small numbers embedded in larger numbers they are introduced to the idea of fractions. At this level the vocabulary of half, halves and quarters rather than the symbols is of greater importance and understanding the equality of two equal parts for halves or four equal parts for quarters. This extends their understanding of partitioning into sharing an object or a set of objects into equal parts.
Partitioning involves sharing a number of objects into equal sized groups (a discrete context) or partitioning an object or shape into the same size pieces (a continuous context)
The continuous context of partitioning underpins the measurement concepts of equal size units, with no gaps or overlaps.

Making two halves the same gives rise to the notion of a balance or equality. Balancing leads to finding shapes that looked balanced. Making shapes that look balanced is a fascinating game with plenty of variety yet it leads to one of the most important concepts of mathematics, the idea of symmetry. The symmetry created by reflection is a valuable tool for discovering properties of shapes.

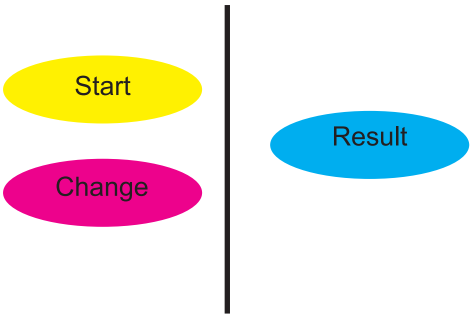
Folding activities are a foundational activity to exploring the idea of angles at a later stage.

Maths Aotearoa Book 1B

Unit 1 - Understanding Addition and Subtraction:

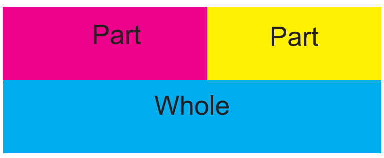
Adding & Taking Away (Structure 1)	Patterns & Relationships (Structure 2)	Complementary Addition & Difference (Structure 3)
Focus on Making 10	Exploring Statistics & Probability Continued through Unit 2	Exploring Position & Orientation Continued through Unit 2

To understand addition and subtraction, students need to have attained the concept known as the cardinal principle. This means they recognise that the last number in the counting sequence tells them how many objects are in the group. Once students achieve this concept they no longer demonstrate a need to count from one when faced with joining two sets together. They can begin a count from any number to find how many altogether.
Addition and subtraction make sense and student will begin to memorise specific facts. It is likely they have already memorised some doubles facts (as met in Maths Aotearoa Level 1a)
This unit focuses on developing a conceptual understanding of addition and subtraction and using the symbols to represent addition and subtraction situation.
There are three different structures for addition and subtraction situations. All structures have three numbers, anyone of which can be the unknown.



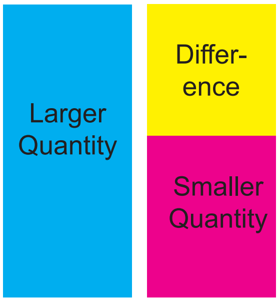
Join: Start amount, Change - the bit being added, Result – the total amount (because addition is commutative the start and the change can be swapped around without affecting the result – it is only for efficiency that we start with the larger number.)

Separate: Start is the largest amount, Change – the bit taken away, Result – the amount left. (Subtraction is not commutative)



This structure leads to the idea that numbers are embedded in other numbers. It is the structure required to support the concept of a family of facts and subtraction being the inverse of addition.
Using number strips or Numicon are an ideal resources for this structure.

Structure 3



This third structure involves the comparison of two quantities. The third quantity does not actually exist. It is the difference between the two amounts.
Additive comparison problems are the most difficult for students to comprehend, possibly because of the non-existence of the third quantity.

All three structures can be represented in word problems and all three structures should be connected together when using physical materials like Numicon, number strips, tens frames etc.
Using only counters tends to lead to the over emphasis of the first (and most simple structure) being addition and subtraction and particularly for subtraction is likely to keep students counting from one as they count out a set of objects, count the ones taken away and then count the ones left.
Students should be encouraged to notice connections between addition and subtraction from the outset.

Mathematics is becoming more representative so a focus must be given to representing mathematical situations through words, pictures and numbers and simple equations. Students are beginning to make sense of symbols as representing mathematical situations.
Being able to draw a picture or model a situation with materials tells you students are able to visualise the problems and they can go on and solve the addition or subtraction by counting.

Using an equation to represent a situation is making more abstract demands. The symbols represent concepts and processes that have been built up through an appropriate range of experiences and dialogue about what is occurring. Mathematical meaning must be acquired before any form of symbolism is introduced.

Early experiences of mathematical symbols, used as a way of presenting work to students with limited reading ability will lead students into learning the symbols as an instruction to do something rather than a way of communicating numeric relationships.
The addition and subtraction symbol are operational symbols. They tell you to perform a specific operation. Most students presented with an equation will use the first addition/subtraction structure to give an answer. The equals sign is then learnt as an operation symbol meaning give the answer rather than as a relationship symbol representing equality of either side of the equation.

Overuse of completion of equations is likely to result in unintended learning consequences. To learn the meaning of the symbols, students need plenty of practice in writing the equations for specific situations most often given in simple word problems.
The problems should be written using familiar language and simple syntax as students need to take responsibility for reading mathematical problems from the outset. As students develop their reading skills, use mathematics as a genre for reading. It requires a high level of comprehension. Provide and assist students to read more complex word problems requiring the use addition and subtraction in all the different structures.
Give students the opportunity to create their own word problems. Students are likely to require assistance in formulating the language to describe the addition or subtraction scenario. As they become more proficient with the language to describe a situation, they begin to reflect a growing understanding of the concepts of addition and subtraction. Evidence of understanding should be reflected in the increasing variety and complexity of their addition and subtraction number stories.

Graphing, pictographs and bar graphs provide a good context for structure three, finding the difference by answering questions like: How many more? How many less? This same structure will be met again in Unit Three where measurement is the context when answering questions like: How much longer? How much shorter?

Unit 2 Larger Numbers and Beginning Multiplication

Sequencing to 100	Number Patterns with Multiples	Equal Grouping & Repeated Addition
Doubles & Halves	Exploring Statistics & Probability Continued from Unit 1	Exploring Position & Orientation Continued from Unit 1

Foundations for multiplicative thinking begin early in a student's learning and are found in counting (focus of this unit) and measuring (focus of unit 3) experiences. In unit 4 of level 1a students were exposed to equal grouping and some of the activities in this unit are a repeat of activities suggested in 1a unit 4.

An understanding of quantity provides the foundations for understanding multiplication. A quantity is a characteristic of objects that can be counted or measured. The number word describes the number portion of quantity. A number on its own does not exist. It can only be described in relation to something – the unit, for example counters. A count is a discrete quantity that answers the question How many? Counting has already begun with counting in ones and now progresses to counting in equal size groups. The unit component of a quantity is essential for solving multiplication type problems. Additive situations most often involve joining two quantities of the same unit (3 lollies and 5 lollies).

In a multiplicative situation the two quantities often refer to different units. (e.g 3 lollies in each of 5 bags). In visualising the problem the correct quantity unit must be attributed to each number. Understanding the quantity unit can be one of the comprehension barriers when reading multiplication type problems, which is why it is important to use meaningful contexts for problems.

Equal grouping problems can be viewed additively as equal additions with the student focusing only on adding the quantities of the same unit. It is important to represent the problems with materials or in pictures to ensure students focus on the quantity unit and not just the number. Students need to learn to co-ordinate the three aspects of a multiplicative situation: Group size, the number of groups and the total amount. Introduce students to a multiplicative representation of equal groups called an array.

Compared to the time taken to develop additive thinking, the time taken to develop multiplicative thinking may take many years. Students normally develop the ability to add and subtract naturally but multiplication is much more complex. This early stage focuses mainly on equal grouping but using vocabulary like twice, three times in every day situation pushes the idea of multiplicative comparison thinking.

This unit explores the idea of counting in equal groups as the beginnings of solving multiplicative type problems. Students can learn to skip count by rote without any understanding that the counting set is an equal group of more than one so being able to recite a skip counting sequence is not evidence of an understanding of equal grouping.
Many students find counting in multiples of two relatively easy, especially up to 10 or 12 as counting sets of objects in this range is a common everyday occurrence. The teen numbers can cause a problem when counting in ones so it is understandable that they can also cause a problem when counting in multiples of two.
At this stage in a student's learning they are more likely to see learning doubles as an addition idea but the word double is a multiplicative word. Unless this word is fully explored with repeated doubling students will thinking doubling is the same as repeated addition. (Repeated halving occurs in unit three when exploring fractions)

The idea of an equal group is used again on a measurement scale (the repeat of identical units). At higher levels these units of repeat will involve multiples (in the decimal system multiples of 10). A scale is also used on the axis of graphs. At higher levels these are likely to involve multiples.

The idea of an equal group of 10 is fundamental to the place value system of numbers. Students need to be able to manipulate the number of groups and the group size of ten. While one group of ten gives the same answer as ten groups of 1, students must maintain the group of ten to be able to use ten as a counting set. (This specific grouping will be revisited in unit four)

Sequencing numbers to one hundred in English requires the learning of 29 different words. The similarity in sound between for example thirteen and thirty leaves many students with confusion. Students' need plenty of practice in rote counting but they also need to be able to image the sequence – having a 0 – 100 number line on display is really helpful. The 100 chart while taking up less space does not provide the visual sequence of how far or near a number is to another number in the sequence.

In order to avoid or rectify a teen/ty confusion, students also need to consider the cardinal aspect of the numbers meaning the size of the number. Number 13 represents 10 + 3 (Additive structure of place value) and 30 represents 3 groups of 10 (multiplicative structure of place value). Students at this point have not considered 10 as representing one group of 10. Introduce the idea and it will be followed up further in unit four.
Te reo Maori supports both the additive structure of place value 10 + 3 tekau ma toru and the multiplicative structure 3 groups of 10 toru tekau

The hundreds chart can begin to show each row as a group of ten and the decade numbers counting the groups of ten. The student must now begin to notice the position of a digit in a number and know whether they are working with a group of one or a group of ten.
A students understanding of a two digit number must expand :
For example: Students need to understand 24 as:
24 is 24 ones (cardinal)
24 is the number after 23, before 25 and between 23 and 25 (ordinal)
24 is 2 tens and 4 ones (Column position - linguistic place value)
24 is 2 groups of ten and 4 groups of 1 (conceptual place value)

Further work on place value specifically is covered in unit four.

Unit 3: Beginning Fractions

Fractions of Shapes	Equal Sharing	Fractions of Numbers
Reflective Symmetry	Exploring Measurement Continued through Unit 4	Exploring 2D & 3D Shapes Continued through Unit 4

The gradual building of fractional knowledge begins with understanding fractions as partitions or divided quantities.
To develop a conceptual understanding of fractions, students need to revise their understanding of partitioning. Students have met partitioning in the context of addition and subtraction where the size of the parts does not matter. Partitioning into equal sized parts is the fundamental concept for understanding fractions, percentages and decimals. The key idea behind partitioning involves dividing a number or objects into equal sized groups (a discrete context) or partitioning an object or shape into same size pieces, (a continuous context). It is important to use a variety of representations for modelling partitioning to ensure students are thinking about the key ideas being taught and not simply memorising images or procedures to solve problems.

Equal sharing is the beginnings of partitioning a quantity or objects (discrete context for fractions)

Students need to begin to move from the counting strategy, (as met in level 1a) where they count out the total number of objects into the required number of groups and then count the number in each group. Thinking must move to foundational multiplicative thinking. Additive thinking does not support equal sharing and fractional thinking. Additive thinking is likely to lead to misconceptions as students continue to apply whole number thinking to fractions.

Repeated subtraction can be used for equal grouping situations where the size of the equal group is known, but in equal sharing the size of the group is unknown; the known is the total and the number to be shared between. While repeated subtraction would give a correct answer is does not model the situation of equal sharing. Equal sharing is mathematically more complex than equal grouping as it requires the foundational multiplicative thinking.
The student needs to consider of the total number of items to be shared, the number to be shared between and the number of items in the equal group received. Like for multiplication, students are being asked to manipulate all three numbers at the same time. Using an array structure (as introduced in unit two) promotes the use of multiplicative thinking by encouraging students to visualise the equal share and the equal groups that make up a set of objects.

Eventually students recognise the multiplicative relationship between the total, the number of groups and the number in each group and are able to use the inverse of multiplication, called division. (For most students this will take many years and lots of multiplicative experiences.)

Representations for the continuous context of fractions come from working with partitioning shapes. Using different shapes to show how some shapes can be halved in more than one way. Some can be halved again to make quarters some cannot. It is just as important to have experiences of not halves and not quarters for students to generalise the need for each part to be equal.

Halving shapes leads to work on exploring reflective symmetry. A repeated halving leads to quarters and two lines of reflective symmetry.

Activities for measuring length provide a visual context for continuous fractions. A measure (e.g. Length) is a particular type of quantity that has a continuous characteristic. Students must come to the understanding each unit of length used to measure a length must be an equal length repeated over and over again, without gap or overlap. With increasing number knowledge and experience of measurement by direct comparison students come to the understanding that another object can be used repeatedly and the number of times it is used can be counted to give a measurement. The essential concept is the object used must be of an equal measure (e.g. length) and the number of objects used must be counted. Measure also requires the understanding that a unit can be broken into a number of smaller equal sized units. At this level smaller units would involve halves and quarters of the unit of measure.
While measurement concepts are the same for all measures, it is the measurement of length that provides the necessary visual context for the concepts. Measuring mass, capacity, angles, temperature and time all require a considerable amount of language which continues to be a focus of measurement experiences.

Understanding measure is connected to the understanding and use of a number line or scale. This is different to a number strip in that the number strip can only represent the natural numbers.

1	2	3	4	5	6	7	8

A number line represents whole number and fractions
The number is represented by a unit length and the number is written at the end of the unit. This allows for a zero and fractions. Students need to come to the understanding that fractions are numbers between the whole numbers.

The words half, quarter and third are not clearly linked to their symbol and therefore naming these early fractions is not obvious to students. It will require plenty of practice to link the words and symbols to the number of equal parts. (Using Te reo Maori names for fractions makes the link much clearer: half - haurua; third -hautoru, quarter hauwha etc).
Fraction words (except half and quarter) are also homonyms being a position in a sequence after first and second. This can add another layer of confusion for students as they are more likely to be familiar with the positional context of the words.
Name fractions like ¾ as “three quarters” and not “three out of four” as this emphasises the use of whole number language. This naming of the fraction only makes sense when using a fraction to represent a proportion.

Unit 4: Beginning Place Value - Unlocking the Number System

Using 10 as a Counting Set	The Importance of a Group of 10	Addition & Subtraction to 20
Using NZ Money	Exploring Measurement Continued from Unit 3	Exploring 2d & 3D Shapes Continued from Unit 3

A multi digit number is the result of adding together the value of the digits in each of the columns. It is therefore important that students are able to name each of the columns. To limit the columns knowledge to just tens and ones is likely to mask the difficulty of dealing with zero. Students may continue to think as 10 as ten groups of one and never as one group of ten, likewise 23 is only 23 groups of one. This thinking is masked as students can often say 2 tens because there are 2 tens in the tens column but when given the number 423 they still only see 2 tens in the number and there is no thinking about groups of ten. Anecdotally teachers often say students seem to get place value with two digit numbers but it all falls over with three digit numbers. When working with two digit numbers periodically extend thinking to the three digit to force a change in thinking using materials and to check they are not solely focused on the digit in the tens column but the number of groups of ten in the whole number.
As students develop an understanding of place value they need to be able to view a two digit number in many different ways
For example the number 23
Twenty three ones
Twenty three as 23
23 as 2 tens and 3 ones (where ten is a counting unit)
23 as 20 + 3
23 as 2 groups of ten and 3 groups of one

The decade numbers do not challenge a student’s thinking about the use of zero and the work at this level is only the beginnings of place value. The biggest difficulty for students at this level is maintaining the number in the group, the number of groups and the total. For place value the groups size is a constant – a group of ten and students need to come to the understanding that groups of ten play a significant part in our number system and is key to the code of how our numbers are represented.

Earlier work on equal grouping should benefit student thinking in equal sized groups and in this unit the thinking is put towards the understanding of the number system – the multiplicative structures and the additive structures within the system.

Sequencing numbers to 100 was met in Unit Two so students should be able to read and write two digit numbers. Listen carefully for continuing confusion between teen and ty numbers and further investigation and modelling of - ty numbers as groups of ten (multiplicative structure) and - teen numbers as 10 + (additive structure) is likely to be necessary for some students.

To understand 30 as 3 groups of ten is the ten times table, counting multiples of ten. Students need to come to the understanding that the basic facts are repeated in each of the columns.
If students know 3 + 3 = 6, then it follows that 30 + 30 = 60. However the language does not support this thinking, students have to understand the symbolic representation of the decade numbers. Three and three is six does not equate to thirty and thirty is sixty. Hundreds and thousands are easier as saying three hundred and three hundred is six hundred is no different to saying three apples and three apples is six apples. Again Te Reo provides a supporting language to the symbolic representation; toru tekau and toru tekau is ono tekau 30 + 30 = 60

Students need to understand there are only ten digits 0 – 9 and where they are placed in a number the digit takes on a specific value; the place value of a digit.
0 – 9 are also referred to as numbers – they are single digit numbers. Numbers like 23 are made up of two digits and not two numbers. This gives rise to the term 2 digit number. A zero in a two digit number can only represent no groups of one. The absence of groups of ten is not represented until you have a three digit number.
Without zero we do not have our number system. Understanding the use of zero in a number is fundamental in reading and writing numbers and understanding the additive structure of the Hindu Arabic system. Students will need to have a rudimentary understanding of zero to make sense of expanding numbers into their component parts. 23 = 20 + 3
Students require many opportunities to make sense of the symbols used to represent numbers.
Understanding the importance of a group of ten is fundamental. Working with equal groups is a fundamental multiplicative idea. The equal group size is based around ten. (The Arabic Hindu number system is a Base 10 system) Two digit place value is just the first group of tens. The regrouping of the groups of ten into another group of ten gives rise to the third column (hundreds). The system is based on continual regrouping of tens or a nesting of groups of ten within groups of ten.

Students do not readily think of one as a group. But the first column represents groups of one. One is known as the multiplicative identity, when you multiply by one you do not change the number. Using the commutative property of multiplication, 5 x 1 = 1 x 5, Students readily see, and all prior experience has taught them to see one group of five but not the five groups of one.

Real life experiences in the classroom will often reinforce the idea that you cannot have a group of 1 as you either work on your own or in a group.

Students need many early experiences to build a foundational understanding of place value. Continuing conceptual understanding of place value through to generalising the number system with whole numbers and decimals will take many years and good teaching and learning experiences.

The addition and subtraction element extends earlier experiences from Unit One and students need further experiences of all three structures of addition and subtraction with result unknown, change unknown and start unknown type problems. (See introduction to unit one for explanation of the three structures).
Equalising problems are also included in this unit as students begin to use the equals symbol as a relationship between two totals.

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